“Advanced Wireless Connectivity"

SigNexT

Spatial Division Multiplex Access (SDMA)

Principles and potential applications

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1 SDMA for Time Division Duplex or Frequency Division Duplex with feedback

1.1 The concept

The concept of SDMA (Space Diversity Multiple Access) is based on *space diversity* idea. According to this idea it is possible to transmit *N* independent data streams, using *Q* transmit antennas ($Q \geq N$), to *N* independent receivers equipped with at least one receive antenna within the same frequency band. And conversely, to transmit *N* independent data streams from *N* independent transmitters while receiving all these data stream in one place using Q ($Q \geq N$) receive antennas within the same frequency band: See Fig. 1.

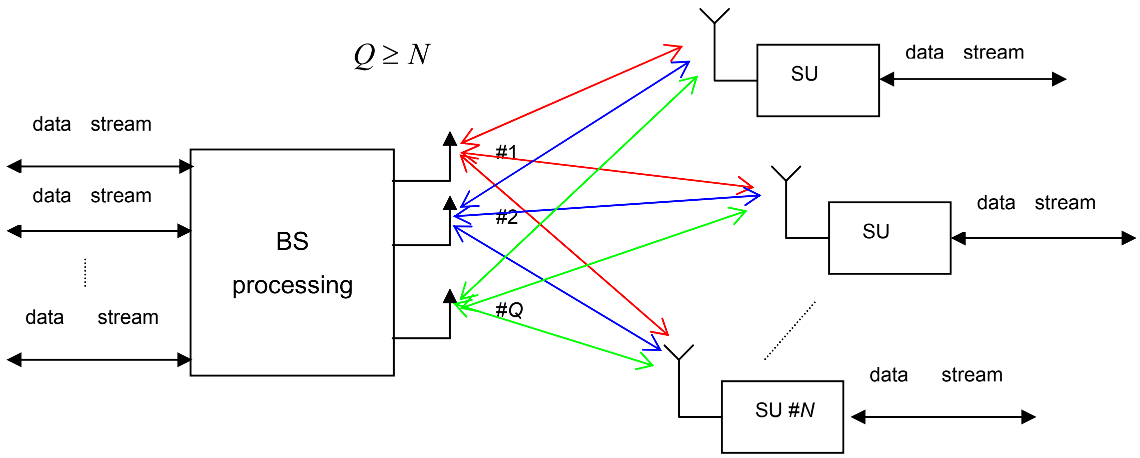

Fig.1

1.1.1 Frequency assignment

As mentioned one frequency band per cell is used. However the frequency band used by neighbor cells must be different. Fig. 2 shows possible frequency assignment that requires only four different frequency bands.

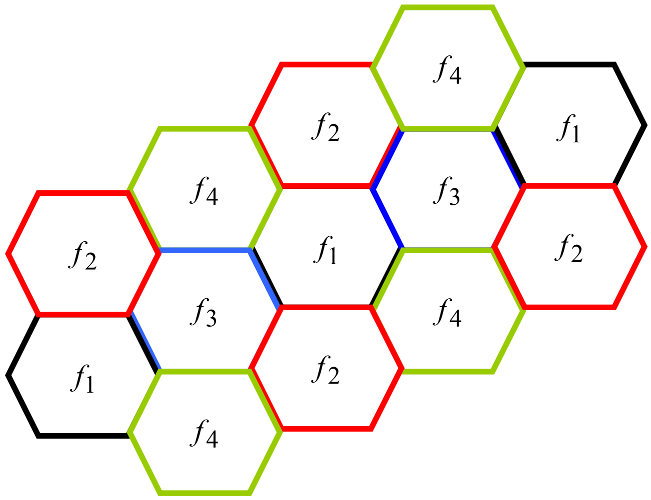

Fig. 2

1.2 Down link algorithm

The baseband model of down link is shown in Fig. 3 The vector $\mathbf{a} = [a_1, a_2, ..., a_N]^T$ represents the symbols to be transmitted at given time instant. The addressee of a_n is the SU # *n.* The matrix $\mathbf{C} = [c_{nq}]$ is the *channel matrix* where the index *n* represents the rx while index *q* represents the tx, i.e. c_{nq} determines the link between *q*-th tx and *n*-th rx. The matrix **H** is a *coding matrix* and is computed by the BS using the **C** estimate. The channel matrix **C** is estimated by SUs: SU # *n* estimates the *n*-th row of **C**. For Frequency Division Duplex (FDD) this information has to be feed back to BS. For Time Division Duplex (TDD) the matrix **C** is estimated during the up-link.

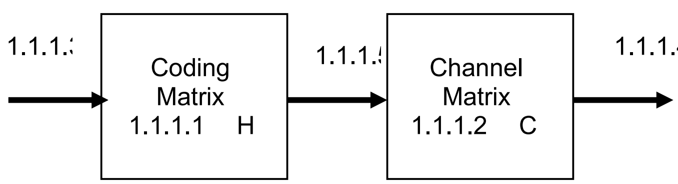

Fig. 3

1.2.1 How to find the coding matirx **H** ?

There are two basic requirements from the matrix **H** :

(**req1**) The *n-th* element of $\mathbf{r} = [r_1, r_2, ..., r_N]^T$ is a sufficient statistic of a_n, i.e.

$$r_n = \alpha_n \cdot a_n + \eta_n, \quad n = 1,2,\ldots,N \qquad (2.1)$$

where α_n is some known coefficient and η_n is a noise. This requirement is just a basic requirement for SDMA.

(**req2.1**) The radiated energy per user equals E_c.

(**req2.2**) The radiated energy per transmitted antenna equals E_c.

The outline of the solution is given in two basic stages:

Stage 1

$$\widetilde{\mathbf{H}} = pinv(\mathbf{C}) = \mathbf{C}^H \left(\mathbf{C}\mathbf{C}^H\right)^{-1} \qquad (2.2)$$

is just the "non-normalized" version of **H** that fulfills req1. For $Q = N$, $\mathbf{b} = \mathbf{C}^{-1}\mathbf{a}$ while for $Q > N$ we have so called *underdetermined* problem [1] with $\mathbf{b} = pinv(\mathbf{C})\mathbf{a}$. This solution guarantees minimal $\|\mathbf{b}\|^2$, i.e., **minimal transmitted total energy.**

Stage 2

Stage 2.1

$$\mathbf{H} = \sqrt{E_c} \cdot \widetilde{\mathbf{H}} \cdot diag(\mathbf{g}) \qquad (2.3)$$

where

$$g_n = \frac{1}{\sqrt{\sum_{q=1}^{Q} \left|\widetilde{h}_{qn}\right|^2}} \qquad (2.4)$$

is the required coding matrix that fulfils re q 2.1.

Stage 2.2

$$\mathbf{H} = \sqrt{E_c} \cdot diag(\mathbf{g}) \cdot \widetilde{\mathbf{H}} \qquad (2.5)$$

where

$$g_q = \frac{1}{\sqrt{\sum_{n=1}^{N} \left|\widetilde{h}_{qn}\right|^2}} \qquad (2.6)$$

is the required coding matrix that fulfils req2.2.

Using (2.3) we have

$$\mathbf{r} = \mathbf{C} \cdot \mathbf{H} \cdot \mathbf{a} + \eta = \sqrt{E_c} \cdot \mathbf{C} \cdot pinv(\mathbf{C}) \cdot diag(\mathbf{g}) \cdot \mathbf{a} + \eta = \sqrt{E_c} \cdot diag(\mathbf{g}) \cdot \mathbf{a} + \eta \qquad (2.7a)$$

or

$$\begin{aligned} r_1 &= \sqrt{E_c} \cdot g_1 \cdot a_1 + \eta_1 \\ r_2 &= \sqrt{E_c} \cdot g_2 \cdot a_2 + \eta_2 \\ &\dots \\ r_N &= \sqrt{E_c} \cdot g_N \cdot a_N + \eta_N \end{aligned} . \qquad (2.7b)$$

$G_n = 20\log_{10} g_n$ represents the SNR loss/gain as seen at SU # *n.*

Conclusion: the matrix $\mathbf{H}$ defined in (2.3) fulfils req.1 and req.2.1.

Using (2.5) we have

$$\mathbf{r} = \mathbf{C} \cdot \mathbf{H} \cdot \mathbf{a} + \eta = \sqrt{E_c} \cdot \mathbf{C} \cdot diag(\mathbf{g}) \cdot pinv(\mathbf{C}) \cdot \mathbf{a} + \eta \qquad (2.8)$$

Unfortunately, as follows from (2.8) req.1 cannot be fulfilled

Conclusion: the matrix $\mathbf{H}$ defined in (2.5) **does not guarantee SDMA** and cannot be used**.**

More detailed block scheme of down link (based on $\mathbf{H}$ defined in (2.3)) is shown in Fig. 4.

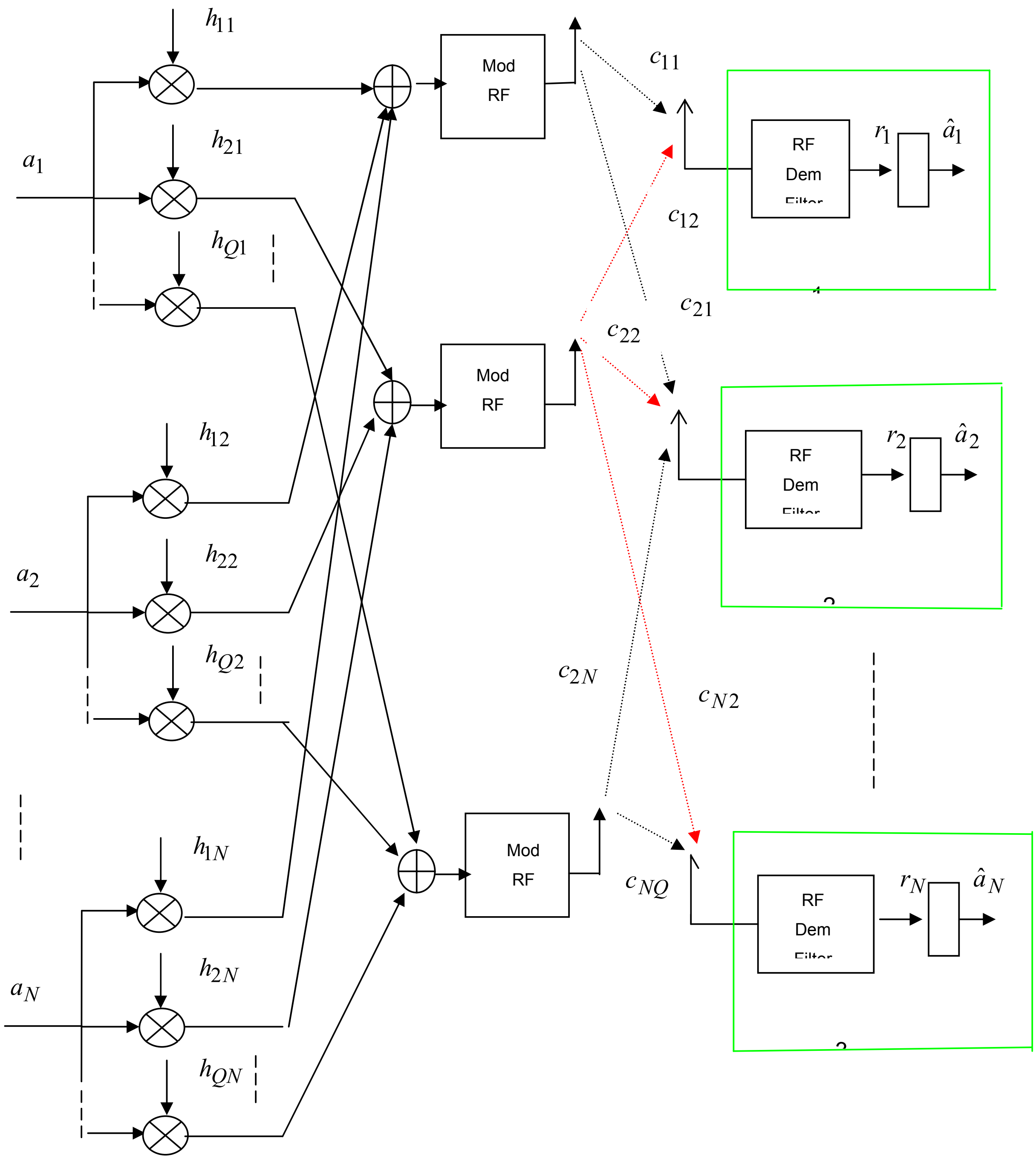

Fig. 4

1.2.2 SNR considerations

From (2.7) and under the assumption that $\mathrm{var}\{\eta_n\} = \sigma_\eta^2$, the value of SNR, seen at SU # *n,* equals to

$$SNR_n = \frac{E_c g_n^2}{\sigma_\eta^2}. \quad (2.9)$$

In fact SNR_n is a conditional SNR given **C**.

Having in mind that g_n^2 is a random variable, the receiving quality depends on its distribution. It is shown for Rayleigh fading channel matrix **C**, by using Monte-Carlo method (unfortunately), that the pdf (probability density function) of g_n^2 equals to the pdf of the following random variable:

$$X = \sum_{l=1}^{L} |c_l|^2 \quad (2.10)$$

for $L = Q - N + 1$. $\{c_l\}_{l=1}^{L}$ used in (2.10) are i.i.d. and have the same distribution as the elements of **C**, i.e., they are mutually independent, complex valued Gaussian random variables with zero mean. Fig. 5 a,b,c,d,e shows the results of Monte-Carlo analysis for 500,000 trials, for $N = 6$ and for $Q = 6,7,8,9,10$ respectively.

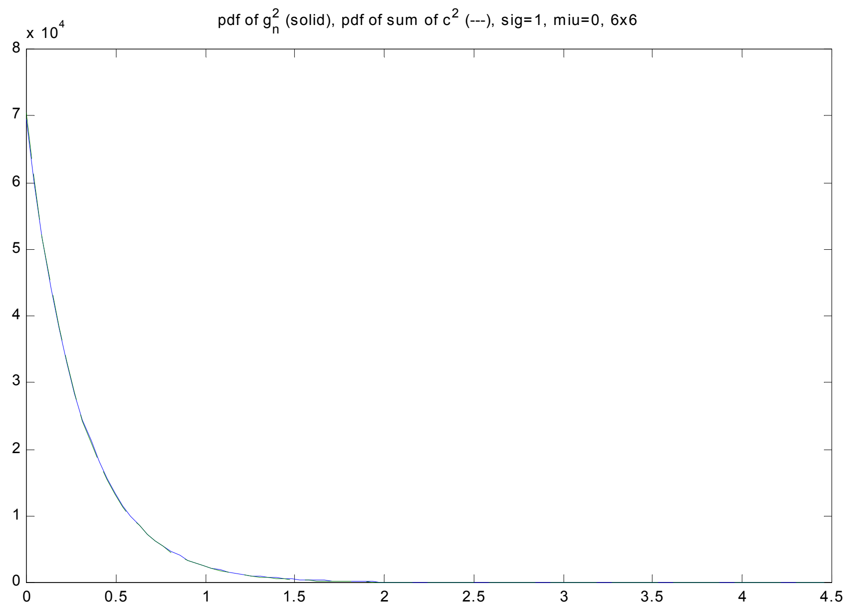

Fig. 5 a

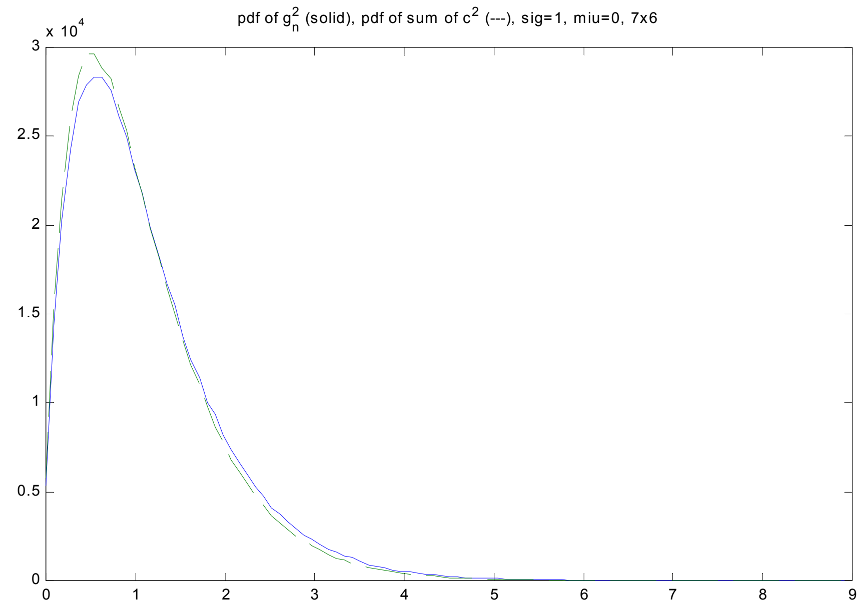
pdf of g_n^2 (solid), pdf of sum of c^2 (---), sig=1, miu=0, 7x6
x 10^4
3
2.5
2
1.5
1
0.5
0
0 1 2 3 4 5 6 7 8 9

Fig. 5 b

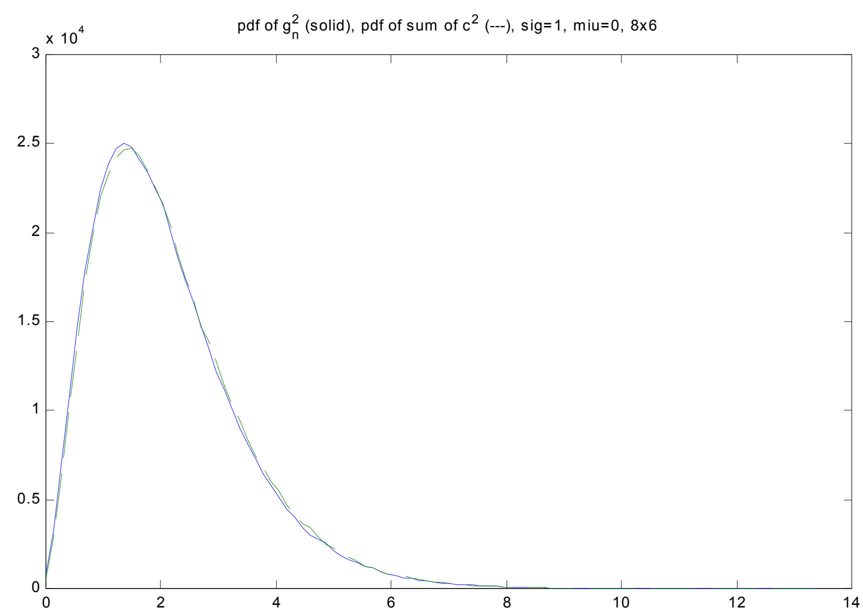
pdf of g_n^2 (solid), pdf of sum of c^2 (---), sig=1, miu=0, 8x6
x 10^4
3
2.5
2
1.5
1
0.5
0
0 2 4 6 8 10 12 14

Fig. 5 c

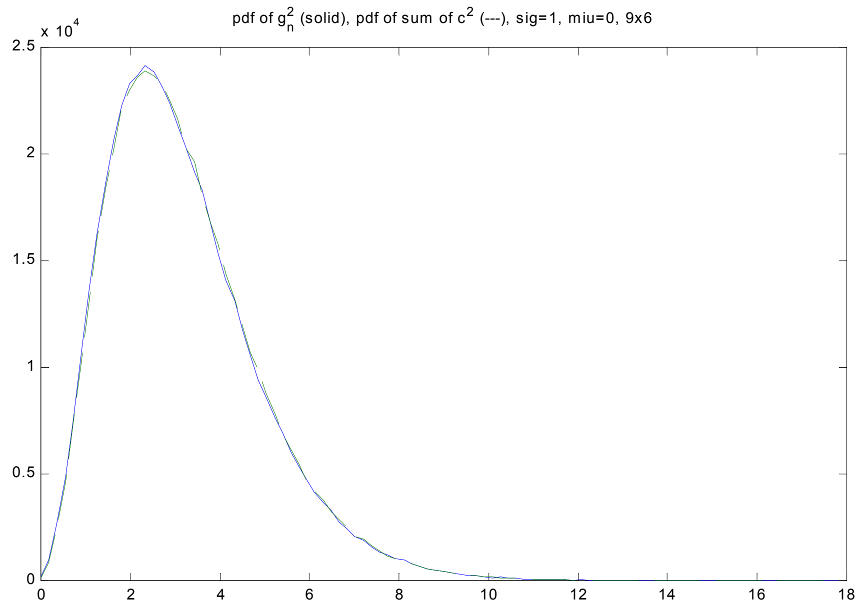
pdf of g_n^2 (solid), pdf of sum of c^2 (---), sig=1, miu=0, 9x6
x 10^4
2.5
2
1.5
1
0.5
0
0
2
4
6
8
10
12
14
16
18

Fig 5 d

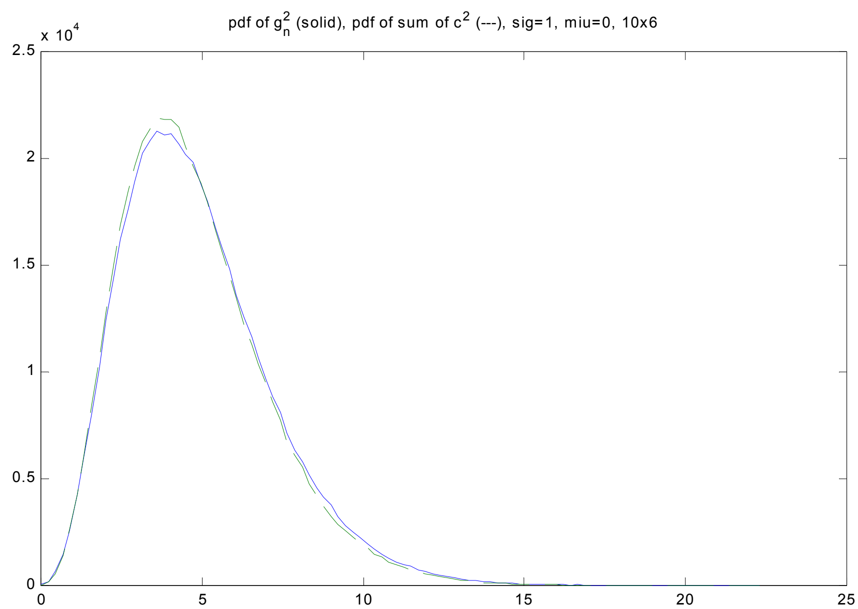
pdf of g_n^2 (solid), pdf of sum of c^2 (---), sig=1, miu=0, 10x6
x 10^4
2.5
2
1.5
1
0.5
0
0
5
10
15
20
25

Fig. 5 e

Consider now a system with one transmit antenna and L receive antennas as shown in Fig. 6

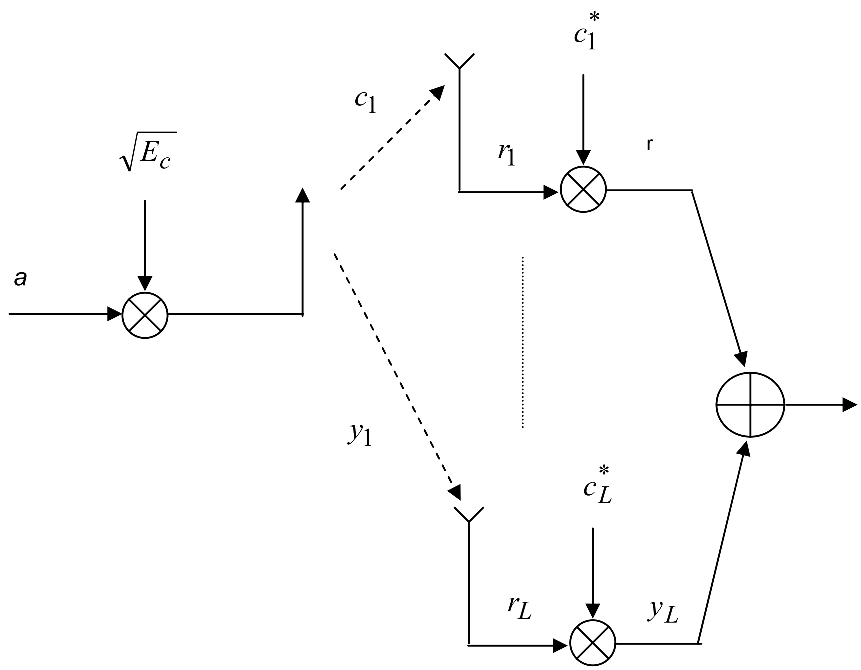

Fig. 6

We have

$$r_l = \sqrt{E_c} \cdot a \cdot c_l + n_l, \quad l = 1,2,...,L \tag{2.11}$$

and therefore

$$y = \sum_{l=1}^{L} \left(\sqrt{E_c} |c_l|^2 a + c_l^* \eta_l \right). \tag{2.12}$$

Assuming that the noises η_l are mutually uncorrelated and have the same distribution, the SNR equals to

$$SNR = \frac{E_c \left(\sum_{l=1}^{L} |c_l|^2 \right)^2}{\sigma_\eta^2 \sum_{l=1}^{L} |c_l|^2} = \frac{E_c X}{\sigma_\eta^2}. \tag{2.13}$$

The system with the SNR as given by (2.13) has *L-th order receive diversity* [3].

1.2.3 Conclusion

Using **H** as given by (2.3) guarantees at each one of SU's effect of receive diversity of the order $L = Q - N + 1$.

1.3 Up link algorithm

It is assumed that all SUs transmits synchronously to BS. The block scheme is shown in Fig. 7

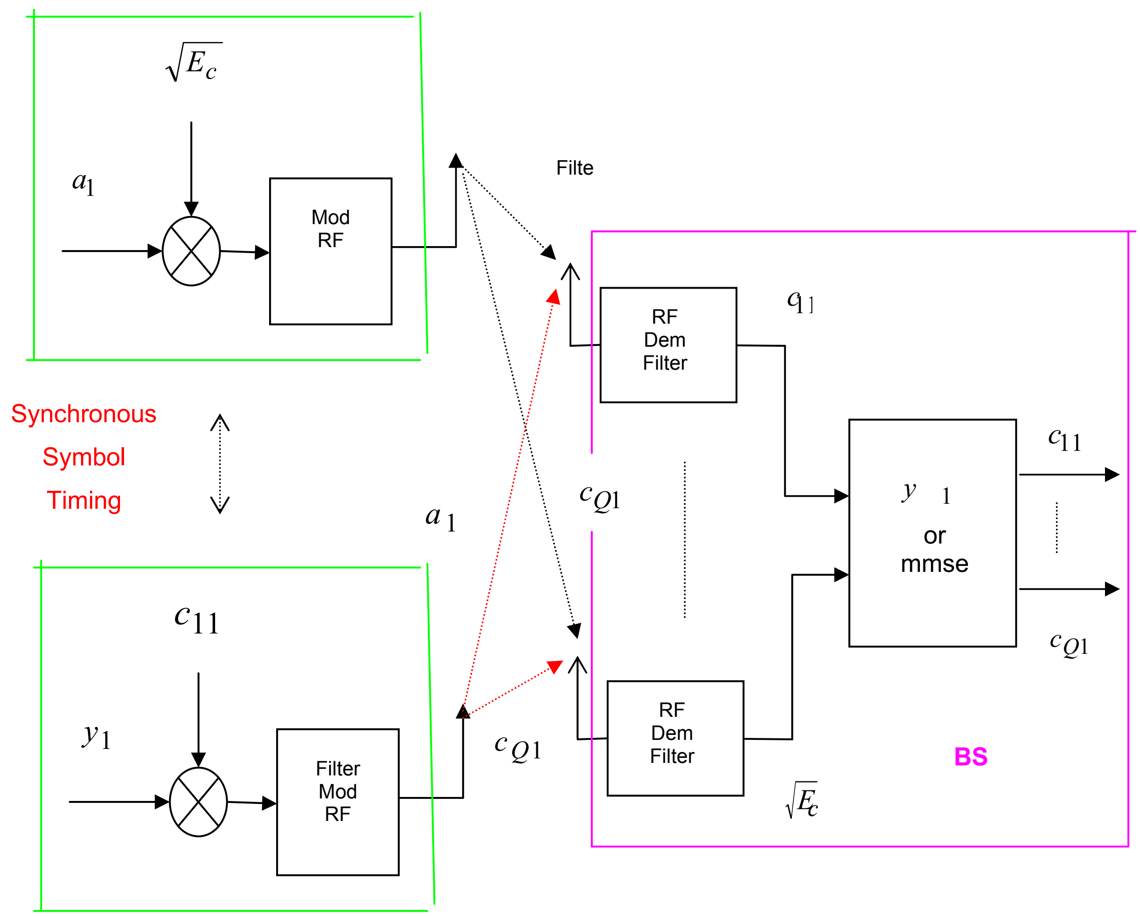

Fig. 7

$$\mathbf{r} = \sqrt{E_c} \cdot \mathbf{C} \cdot \mathbf{a} + \boldsymbol{\eta} . \qquad (3.1)$$

1.3.1 3.1 Pseudo-inverse

Let **H** be the pseudo-inverse of **C.** The matrix **C** is $Q \times N$ with $Q \geq N$ therefore

$$\mathbf{H} = \left(\mathbf{C}^H \mathbf{C}\right)^{-1} \mathbf{C}^H . \qquad (3.2)$$

Applying **H** to **r** yields

$$\mathbf{y} = \mathbf{H} \cdot \mathbf{r} = \mathbf{H} \cdot \left(\sqrt{E_c} \cdot \mathbf{C} \cdot \mathbf{a} + \boldsymbol{\eta}\right) = \sqrt{E_c} \cdot \mathbf{a} + \mathbf{H} \cdot \boldsymbol{\eta} = \sqrt{E_c} \cdot \mathbf{a} + \mathbf{v} . \qquad (3.3)$$

It is known [1] that the matrix **H** (*overdetermined* problem) minimizes $\|\mathbf{v}\|^2$, i.e. the **noise has minimal energy.**

Assuming that $\mathrm{var}\{\eta_n\} = \sigma_\eta^2$, let us find the conditional covariance matrix given **H** (or **C**) of $\mathbf{v}$

$$\mathbf{R}_\nu = E\{\mathbf{v}\cdot\mathbf{v}^H|\mathbf{H}\} = E\{\mathbf{H}\cdot\boldsymbol{\eta}\cdot\boldsymbol{\eta}^H\cdot\mathbf{H}^H\} = \sigma_\eta^2\cdot\mathbf{H}\cdot\mathbf{H}^H. \quad (3.4)$$

Define

$$\mathbf{f} = diag\left(\mathbf{H}\cdot\mathbf{H}^H\right) \quad (3.5)$$

and the instantaneous signal to noise ratio seen at the output # *n*

$$SNR_n = \frac{\frac{1}{f_n}E_c}{\sigma_\eta^2}. \quad (3.6)$$

Having in mind that $\frac{1}{f_n}$ is a random variable, the receiving quality depends on its distribution. It is shown for Rayleigh fading channel matrix **C**, by using Monte-Carlo method (unfortunately), that the pdf (probability density function) of $\frac{1}{f_n}$ equals to the pdf of *X* as given in (2.10). Results shown in Fig. 2.3 a,b,c,d,e are also applicable for $\frac{1}{f_n}$ instead of g_n^2.

1.3.2 Conclusion

Using **H** as given by (3.2) guarantees at each one of the outputs of BS effect of receive diversity of the order $L = Q - N + 1$.

For ML decision **y** has to be supplied to a slicer.

The ML algorithm described above performs just *linear nulling*. Superior performance is obtained if nonlinear techniques are used. One particularly attractive nonlinear alternative is to exploit the timing synchronism inherent in the system model and use *symbol cancellation* as well as linear nulling [2]. Such algorithm is somewhat analogous to decision equalization.

1.4 MMSE

For the case of soft decision the MMSE estimation of **a** is applied prior to use of sequential demodulator:

$$\hat{\mathbf{a}}_{mmse} = \mathbf{C}^T\left(\mathbf{C}\mathbf{C}^T + \frac{1}{SNR}\mathbf{I}\right)^{-1}\mathbf{r}, \quad (3.7)$$

where

$$SNR = \frac{E_c}{\sigma_\eta^2}. \quad (3.8)$$

Note: the formula (3.7) is valid for real case.

2 TDMA/SDMA for FDD without feedback.

2.1 The concept

Consider a case of down link and when the estimate of the channel matrix **C** is not known at the BS due to lack of feedback to transport the estimate of **C** back to the BS. For example, if the system works in FDD. In up link, however the BS acts as a receiver and therefore it estimates the matrix **C** "at home".

Having such situation it is proposed to implement TDMA in down link using space-time block coding [4,5] which make it possible to use Q antennas of the BS in optimal (for Q=2) or quasi-optimal way (for Q=4,8). On the other hand, in up link a SDMA may be used, as it was described in the part A.

2.1.1 Down link algorithm

As mentioned, TDMA is used which means that at a given time slot the BS transmits data destined to one SU only. Fig. 8 illustrates such situation: at a given time slot, BS transmits data destined to BS #2.

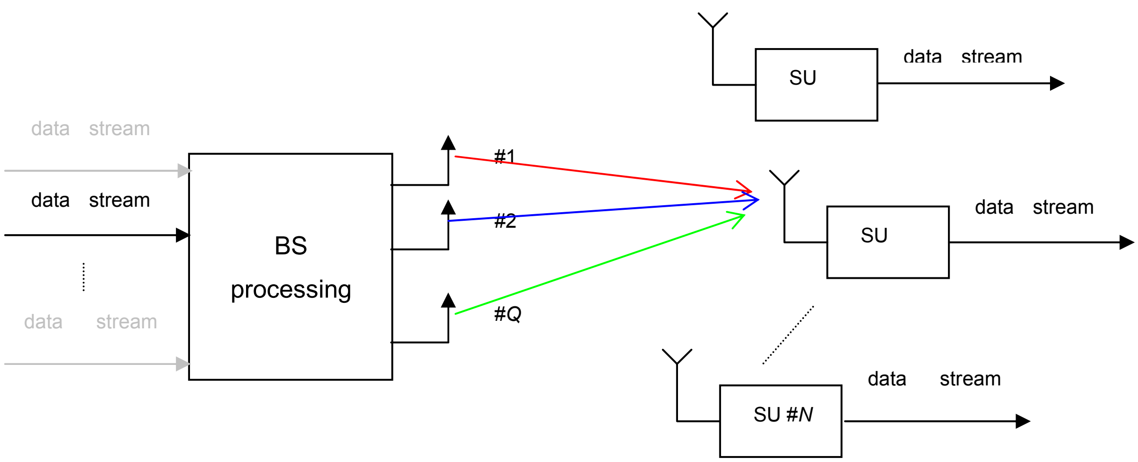

Fig. 8

It is suggested that BS will use a space-time block coding firstly proposed in [4] for $Q=2$ which is fully orthogonal and therefore optimal. This concept was expanded later for $Q=4$ and $Q=8$, resulting with quasi-orthogonal code [5].

2.1.1.1 Description of space-time block coding

2.1.1.1.1 $Q=2$

The input symbols $a(k)$ are gathered into groups of two symbols each $\{b_1(k), b_2(k)\}$, where

$$b_1(k) = a(k-1) \tag{5.1}$$

$$b_2(k) = a(k) \tag{5.2}$$

$$b_1(k+1) = -a^*(k) \tag{5.3}$$

$$b_2(k+1) = a^*(k-1) \qquad (5.4)$$

Fig. 5.2 shows the time diagram that explains the nature of block coding.

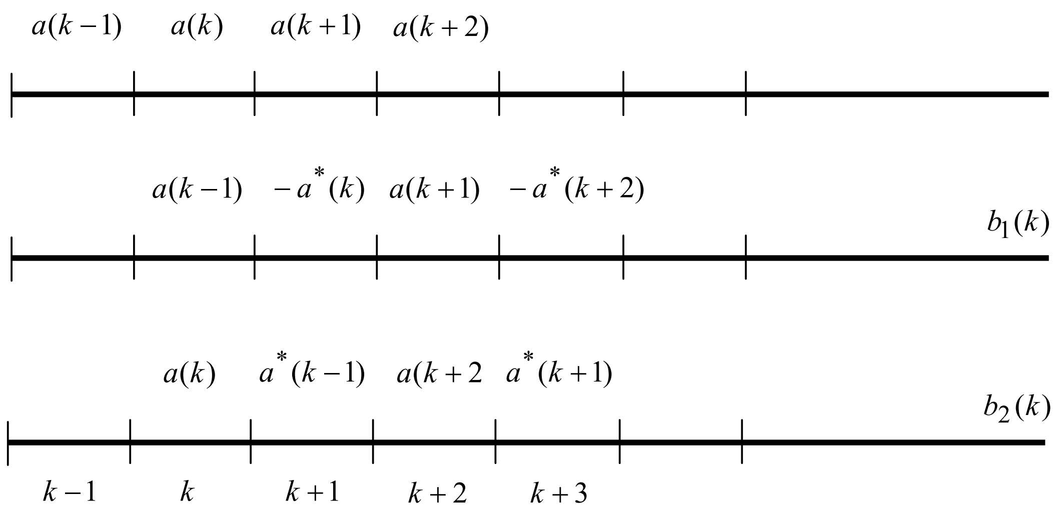

Fig. 5.2

The block diagram of a system based on space-time block coding is shown in Fig. 9.

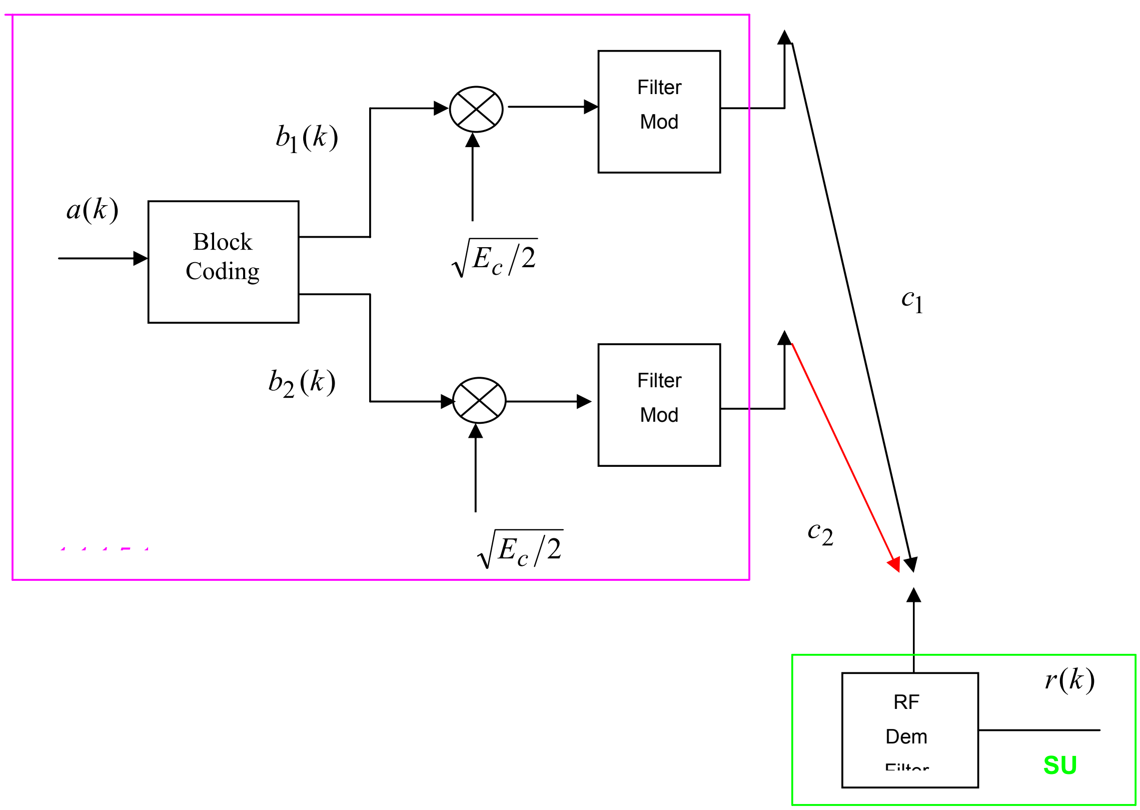

Fig. 9

The signals received by SU during the time slots *k* and $k+1$ are

$$\begin{aligned}
& r(k) = \sqrt{E_c/2} \cdot c_1 b_1(k) + \sqrt{E_c/2} \cdot c_2 b_2(k) + \eta(k) = \\
& \sqrt{E_c/2} \cdot c_1 a(k-1) + \sqrt{E_c/2} \cdot c_2 a(k) + \eta(k) \\
& r(k+1) = \sqrt{E_c/2} \cdot c_1 b_1(k+1) + \sqrt{E_c/2} \cdot c_2 b_2(k+1) + \eta(k+1) = \\
& -\sqrt{E_c/2} \cdot c_1 a^*(k) + \sqrt{E_c/2} \cdot c_2 a^*(k-1) + \eta(k+1)
\end{aligned}, \tag{5.5}$$

Here it is assumed that c_1 and c_2 are constant at least during the time slots *k* and $k+1$.

Define:

$$\mathbf{r}_k \stackrel{\Delta}{=} \left[r(k), r^*(k+1)\right]^T; \tag{5.6}$$

$$\mathbf{a}_k \stackrel{\Delta}{=} \left[a(k-1), a(k)\right]^T; \tag{5.7}$$

$$\boldsymbol{\eta}_k \stackrel{\Delta}{=} \left[\eta(k), \eta^*(k+1)\right]^T; \tag{5.8}$$

$$\mathbf{C}_k \stackrel{\Delta}{=} \begin{bmatrix} c_1 & c_2 \\ c_2^* & -c_1^* \end{bmatrix}; \tag{5.9}$$

$$\mathbf{C}_k^* \stackrel{\Delta}{=} \begin{bmatrix} c_1^* & c_2 \\ c_2^* & -c_1 \end{bmatrix}. \tag{5.10}$$

We have

$$\mathbf{r}_k = \sqrt{E_c/2} \cdot \mathbf{C}_k \cdot \mathbf{a}_k + \boldsymbol{\eta}_k, \tag{5.11}$$

$$\mathbf{C}_k^* \cdot \mathbf{C}_k = \rho_k \cdot \mathbf{I}, \tag{5.12}$$

where

$$\rho_k = |c_1|^2 + |c_2|^2. \tag{5.13}$$

The following may be obtained

$$\tilde{\mathbf{r}}_k \overset{\Delta}{=} \mathbf{C}_k^* \cdot \mathbf{r}_k = \mathbf{C}_k^* \left(\sqrt{E_c/2} \cdot \mathbf{C}_k \cdot \mathbf{a}_k + \boldsymbol{\eta}_k \right) = \sqrt{E_c/2} \cdot \rho_k \cdot \mathbf{a}_k + \tilde{\boldsymbol{\eta}}_k . \qquad (5.14)$$

The SNR equals to

$$SNR = \frac{E_c}{2} \frac{\left(|c_1|^2 + |c_2|^2 \right)^2}{\left(|c_1|^2 + |c_2|^2 \right) \sigma_\eta^2} = \frac{E_c}{2} \frac{\left(|c_1|^2 + |c_2|^2 \right)}{\sigma_\eta^2} \qquad (5.15)$$

From (5.15) it follows that this scheme provides 2-path diversity gain as compared to 1x1 scenario for which

$$r_k = \sqrt{E_c} a_k c_k + \eta_k \qquad (5.16)$$

yielding

$$SNR = \frac{E_c |c_k|^2}{\sigma_\eta^2} \qquad (5.17)$$

The ML decision rule is

$$\hat{\mathbf{a}}_k = \arg\min_{\mathbf{a}_k} \left\| \tilde{\mathbf{r}}_k - \rho_k \cdot \mathbf{a}_k \right\|^2 \qquad (5.18)$$

2.1.2 Implementation problems for large Q

Time space coding in general and block coding in particular increase the capacity of the system. Large *Q* implies large capacity by increasing the average SNR at the receiver. For given and constant bandwidth, the high SNR may be translated to high data rate by increasing the number of points of the constellation more than 256-QAM, the largest constellation used in single antenna system. If for example the SNR increased, due to use block code with $Q = 8$, to justify use a constellation with 12bits/symbol, i.e., 4096-QAM , it may be very difficult to implement such constellation.

2.2 Up-link algorithm

See section 1.3

3 C. Potential applications

- Cellular telephony

- Wireless access

- Pico-cell

- HDTV wireless for home usage

- WiFi

4 References

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